

Efficacy of the isospin-symmetry-breaking correction in Fermi beta decay

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In the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ud} from superallowed Fermi beta decay, a crucial requirement is the application of nuclear-structure-dependent theoretical corrections. True, these corrections are small – of the order of 1% or less – but their evaluation is subject to some uncertainty. One of the most important of these corrections is the isospin-symmetry-breaking correction, δ_C , which is wholly nuclear-structure dependent. It is defined as

$$M_F^2 = M_0^2 (1 - \delta_C), \quad (1)$$

where M_F is the Fermi matrix element and M_0 is its value in the isospin-symmetry limit. The most widely used calculations of δ_C are those of Towner and Hardy (TH) [1] but those calculations have recently been criticized by Miller and Schwenk (MS) [2], who claim that they are based on a formally incorrect interpretation of the isospin operator. Although MS claim that this “incorrect” usage must have led to incorrect results for δ_C , they do not produce any “exact” calculations with which to compare. Instead they proceed to make significant model-dependent assumptions of their own, from which they conclude – without numerical results – that the omissions from TH must be significant in magnitude. In our opinion, their model is considerably less sophisticated than ours and has not been independently corroborated by experimental data as ours has. Thus, if there is indeed any omission in the TH corrections, there is currently no valid estimate of the importance of that omission.

For now, the best that can be done to validate any set of calculated corrections is to test their efficacy in achieving agreement with the basic tenets of weak-interaction theory, particularly the conserved vector current (CVC) hypothesis, but also the unitarity of the CKM matrix. It is in tests such as these that the TH calculations perform astonishingly well, strongly suggesting that the putative omissions, if any, must have a negligible effect.

A variety of different models have been used in the past for the isospin-symmetry-breaking correction. They are:

- **Shell model – Saxon-Woods (SM-SW).** This is the model of TH, in which proton and neutron radial functions are taken as eigenfunctions of a Saxon-Woods potential whose parameters are adjusted to match experimental separation energies.
- **Shell model – Hartree-Fock (SM-HF).** This is similar to SM-SW except that the radial functions are taken to be eigenfunctions of a mean-field Skyrme-Hartree-Fock potential. First proposed by Ormand and Brown [3], their protocol was recently altered by Hardy and Towner [4] to ensure that the proton mean field had the asymptotically correct form.
- **Hartree-Fock -- Random Phase Approximation (HF-RPA).** A Skyrme-Hartree-Fock calculation is performed for the even-even A -body system (the decaying state when $T_Z = -1$,

the daughter state when $T_Z = 0$). The odd-odd nucleus is then treated as a particle-hole excitation built on the even-even Hartree-Fock state. The particle-hole spectrum is computed in the RPA with charge-dependent interactions. First calculations of this type were performed by Sagawa, van Giai and Suzuki [5]. Recently, they were extended by Liang, van Giai and Meng [6] who replaced Skyrme zero-range interactions by finite-range meson-exchange potentials. They performed relativistic Hartree-Fock (RHF-RPA) calculations and relativistic Hartree-only (RH-RPA) calculations with density-dependent meson-nucleon couplings and non-local interactions.

- **Isovector Monopole Resonance (IVMR).** A particle-hole picture was also envisaged by Auerbach [7] in which isospin-symmetry breaking in the parent and daughter states of beta decay is attributed to the difference in their couplings to the giant isovector monopole resonance. To obtain numerical estimates, Auerbach appealed to a number of “gross” models discussed in ref [8]. Each model enabled him to obtain a simple expression for δ_C as a function of the mass number A . As an example, his expression in the microscopic model is

$$\delta_C = 18.0 \times 10^{-7} A^{5/3}. \quad (2)$$

- **Damgaard Model.** First estimates of δ_C were provided by Damgaard [9], who expanded the proton radial function in terms of a complete set of neutron oscillator functions. The set comprises states of the same orbital angular momentum, ℓ , but differing in the number of radial nodes, n . Most of the mixing was with the state with one more radial node. Attributing this mixing to the Coulomb force, Damgaard derived

$$\delta_C = 0.2645 Z^2 A^{-2/3} (n+1)(n + \ell + 3/2), \quad (3)$$

which exhibits a general behavior $\delta_C \propto A^{4/3}$ with some shell structure superimposed through the choice of oscillator quantum numbers n and ℓ . In particular, a proton radial function with one radial node gets a factor of two enhancement in its δ_C value over that with no radial nodes, simply from the factor $(n+1)$ in Eq. (3). Such factors are absent in the formulae of Auerbach [7].

The computation of isospin-symmetry breaking in Fermi beta decay is patently model dependent. So on what grounds can one assert that one model is better than another? The usual way forward in such situations is to appeal to experiment, but in this case experiment does not directly measure the correction δ_C . However, experiment has led to precise values for the ft values of a large number of superallowed transitions – 13 in all – and, if we assume that both CKM unitarity and CVC are satisfied, we can convert those experimental ft values into experimental values for δ_C and compare the results with each calculation in turn. To be more specific, the application of our two assumptions is described as follows:

- *The conserved vector current (CVC) hypothesis is correct.* Under this hypothesis, the corrected Ft values in superallowed Fermi beta decay must be the same for all nuclei with the same isospin assignment. The relation between the corrected Ft value and the experimental ft value is

$$Ft = ft (1 + \delta_R) (1 - \delta_C) , \quad (4)$$

where δ_R is the nucleus-dependent part of the radiative correction and δ_C is the isospin-symmetry-breaking correction. To date, 13 nuclei ranging from ^{10}C to ^{74}Rb have superallowed transitions with ft values measured to an accuracy of $\pm 0.4\%$ or better. Thus we can obtain an “experimental” value of δ_C from the relation

$$(1 - \delta_C) = [Ft]_{\text{av}} / \{ft(1 + \delta_R)\} \quad (5)$$

using ft values from the most recent data survey [4] and the calculated radiative corrections ($\delta_R = \delta_R' + \delta_{\text{NS}}$) from ref. [1]. The value we use for $[Ft]_{\text{av}}$ follows from our second assumption.

- *The CKM matrix is unitary.* The sum of the squares of the top-row elements of the matrix is assumed to be exactly equal to one. The value of V_{us} obtained from the analysis of kaon-decay data, 0.22521(94) [10], and V_{ub} from the Particle Data Group, 0.00393(36) [11], are also assumed to be correct. Under these conditions the value of $|V_{ud}|^2$ is given by:

$$|V_{ud}|^2 = 1 - |V_{us}|^2 - |V_{ub}|^2 = 0.94927 \pm 0.00042 . \quad (6)$$

Further, $|V_{ud}|^2$ is inversely proportional to $[Ft]_{\text{av}}$, the average corrected ft value. Consequently we can write

$$[Ft]_{\text{av}} = (2915.64 \pm 1.08) / |V_{ud}|^2 = 3071.47 \pm 1.78 \text{ s} , \quad (7)$$

the value we use in evaluating Eq. (5).

Thus, we have a set of “experimental” δ_C values, compared to which the various theoretical calculations can be assessed. In Fig. 1 we plot these “experimental” δ_C values as points with error bars, together with theoretical values from the different models available. The success of each model can be

judged by the quality of the fit. To quantify this we evaluate the χ^2 per degree of freedom using the equation

$$\frac{\chi^2}{\nu} = \sum_{i=1}^N w_i (\delta_C^{theo}(i) - \delta_C^{exp}(i))^2 / (N-1) \quad (8)$$

where N is the number of cases computed with a particular theory (*e.g.* the authors of HF-RPA only compute δ_C for 9 of the 13 cases). The weights, w_i , are taken to be $1/\sigma_i^2$, where σ_i is the standard

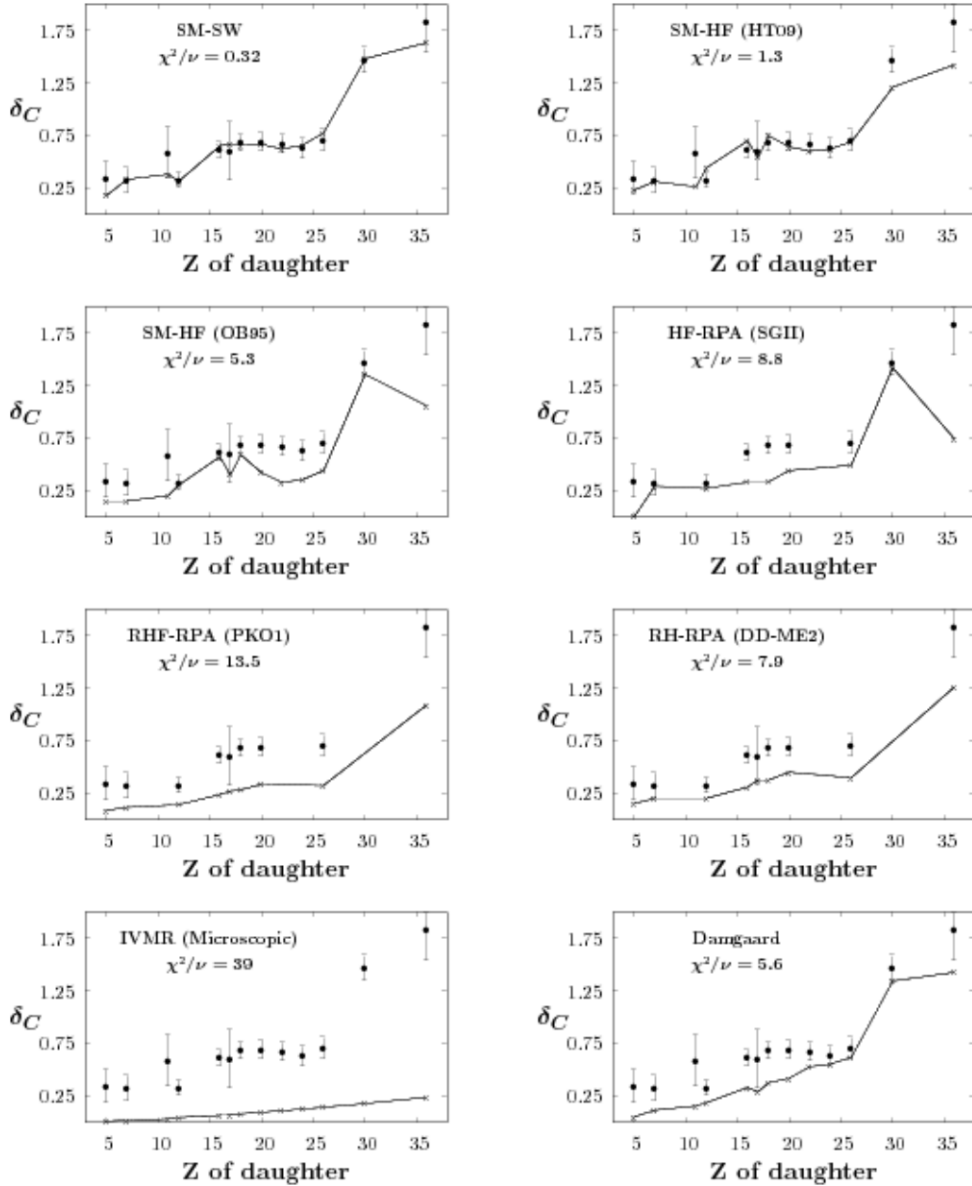


FIG. 1. Isospin-symmetry breaking correction, δ_C , in percent units as a function of the charge number, Z , for the daughter nucleus. “Experimental” values of δ_C (points with error bars) are those required of the theory if the experimental data is to satisfy the CVC hypothesis and the unitarity of the CKM matrix. The references for each calculation are as follows: SM-SW [1], SM-HF (HT09) [4], SM-HF (OB95) [3], HF-RPA (SGII) [5], RHF-RPA (PK01) [6], RH-RPA (DD-ME2) [6], IVMR (Microscopic) [7,8], Damgaard [9].

deviation assigned to the “experimental” δ_C value. It is clear from the figure that the best agreement comes from the shell model, with Saxon-Woods radial functions doing better than Hartree-Fock radial functions. The HF-RPA and IVMR particle-hole models are generally under-predicting the correction, particularly in the heavier nuclei.

Obviously, if superallowed beta decay is to continue to be used to test the unitarity of the CKM matrix, one has to accept the possibility that the “true” correction terms might lead to Ft values that do not satisfy the unitarity condition. However, if they lead to Ft values that change from transition to transition – thus violating CVC – one would not even be justified in extracting a value for V_{ud} at all, let alone using that value to test CKM unitarity. In judging the results of Fig. 1 in that context, we might consider that the most definitive test of the isospin-symmetry-breaking corrections is whether they fit the variations in the “experimental” values from one Z value to another as required by CVC, irrespective of a possible overall additive constant, which would be the effect of a violation of CKM unitarity. Using this criterion, the shell-model results remain the best, but the RHF-RPA (PK01) model ranks as a close second.

With no numerical calculations to check, the Miller and Schwenk claims [2] cannot be tested at all, but with respect to the many models that actually can be checked, the shell-model calculations of Towner and Hardy [1, 4] stand up very well.

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